Problems

Dynamics

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These problems have several purposes: to help you understand the physical principles taught in the course lectures, to apply these in a range of circumstances, and to develop your problem-solving and mathematical-modelling skills. Although one important aim is to get the correct numerical results — which are given on the last page — it is also important that you use these problems to think about the underlying physics, and how you are making mathematical models to solve problems. These problems are probably quite different in style to those you have seen before, and also probably more difficult, and therefore you will take time to learn the skills required to answer them efficiently. Talking to others, in particular your fellow students and your supervisor, is a valuable way to explore different approaches to answering problems.

Also note that many of the questions posed here are structured into smaller parts, to help guide you through what might be quite a complex question that you might not see how to do straight away. However, one of the skills you will need to develop is how to divide up questions yourself, and there are some questions in which you will have to define your own path to solve the problem.

There are 24 problems in all — you should attempt about six per week, but be guided by your supervisor.

Overleaf are a set of guidelines which you should adopt when you tackle **any** physics problem.

For those who are interested, there are a few **strictly optional** additional problems — less structured and usually of a slightly more mathematical nature — which are available via the teaching pages on the Physics website:

`http://www.phy.cam.ac.uk/students/teaching` .

(Click on 'Information for Students' and then, on the Resources for Students page, select 'Electronic copies of all handouts'.)

A list of questions from past Tripos papers relating to the material covered in this part of the course is also available on the above website. Again, these questions are **strictly optional**, but you may find them helpful for the purposes of revision.

Course Synopsis

Introduction to university physics: role of experiment; mathematical models; dimensional analysis; tackling physics problems.

Experimental physics: random and systematic errors; Gaussian probability distribution; mean, standard deviation, error in the mean; errors in functions of a single variable, combining errors in two variables; examples of techniques for dealing with systematic errors; graphs.

Dynamics: *Concept of a force:* tendency to produce motion; forces as vectors; action and reaction; friction. *Calculus in physics:* use of integration. *Work:* potential energy; stable and unstable equilibrium. *Kinematics:* displacement, speed, velocity, acceleration. *Newton's laws of motion:* equations of motion. *Kinetic energy:* concept and definition; principle of conservation of energy. *Linear momentum:* concept and definition; conservation of linear momentum; rockets; elastic and inelastic collisions; impulse of a force. *Frames of reference:* relative velocities, inertial frames of reference, zero-momentum frame, collisions.

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How to solve physics problems

Here is some general advice which you should apply to almost every physics problem you tackle. Now is the time to develop techniques which make the problems easier to understand and which can provide you with a framework within which to think about physics.

1. **Almost always draw a diagram**

   A diagram can help to clarify your thoughts. Use it to define the symbols you need (see 3 below). Make it big enough and be tidy.

2. **Think about the physics**

   Ask yourself what is going on, and write it down in words. Try to *understand* the problem qualitatively before writing down any equations. Do not just write down equations!

3. **Stay in symbols until the end**

   At school you may have been taught to make calculations numerically rather than algebraically. However, you usually give yourself a big advantage if you delay substitution of numerical values until the last line as it enables you to check dimensions at every stage, and quantities often cancel before the last line. An exception to this rule arises where some terms are dimensionless factors which are simple fractions.

4. **Check the dimensions**

   Think about the dimensions of each quantity as you write it down. You will find this a discipline which helps enormously to avoid errors and guides understanding. Make sure that the *dimensions* of your final algebraic equation match on each side before you make a numerical substitution. When making a numerical substitution, write down the *units* of your answer, e.g. 4.97 J kg^{-1}.

5. **Does the answer make sense?**

   You will probably have an idea of what looks about right, and what is clearly wrong. Many mistakes are simple arithmetic errors involving powers of ten. If in doubt, check your substitutions. When giving the final numerical answer to a problem use a sensible number of significant figures.

Introduction

1: The 'Planck time' is a combination of the fundamental constants h, G and c, in the form $h^{\alpha}G^{\beta}c^{\gamma}$. G has units of N m^2 kg^{-2}; h has units of J s; c has units of m s^{-1}. Use dimensional analysis to derive the values of α, β and γ.

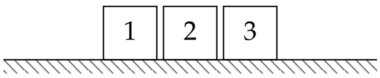
2: This is an illustration of the difference in style between A-level physics questions and university physics questions.

2.1 *On p. 4 is a question taken from the OCR paper 2824,* Forces, fields and energy, *set in June 2008. You will see that it is set out on two sides of A4, is divided into small sections which you are guided through step-by-step, diagrams and important algebraic variables are provided for you and you are required to carry out almost no algebraic manipulation.*

Work through the A-level question on p. 4 as set.

2.2 *The corresponding question written in university physics style is given below. It consists of just a few sentences and a single diagram and requires more in the way of abstract thought, visualisation of the problem, and planning your own route through it.*
Work through this question as now set in university style. Make sure you follow the advice given on p. 2. Check that your algebraic answers agree in case (b)(i) with the solutions given in the original A-level question and in case (b)(ii) with your qualitative conclusions to the original A-level question.

Three identical perfectly elastic cubes are placed on a frictionless horizontal surface as shown.



(a) The three cubes are in contact and a steady horizontal force P is applied to the left-hand end surface of cube 1; find the magnitudes of the forces on cube 2 from cubes 1 and 3.
(b) Cube 1 is moved to the left and projected towards cubes 2 and 3 which are stationary. Find the speeds of all three cubes after the collision if before the collision (i) cubes 2 and 3 are separate but nearly touching or (ii) cubes 2 and 3 are glued together.

Answer **all** the questions.

1 This question is about the interactions between three identical perfectly elastic solid cubes.

(a) Fig. 1.1 shows three numbered cubes each of mass m placed in contact on a frictionless horizontal surface. A steady horizontal force P is applied to the end surface of cube **1**.

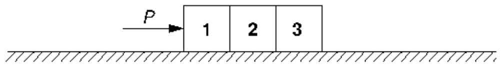


Fig. 1.1

(i) Show that the acceleration of cube **3** is $\frac{P}{3m}$.

[1]

(ii) Write down an expression for the resultant force F_3 on cube **3**.

F_3 = ...[1]

(iii) Write down expressions in terms of P for

the acceleration a_2 of cube **2**

a_2 = ...

the resultant force F_2 on cube **2**.

F_2 = ...[2]

(iv) Hence write down an expression for the magnitude of the force applied by

cube **3** on cube **2**, F_{32}

F_{32} = ...

cube **1** on cube **2**, F_{12}.

F_{12} = ...[2]

(b) Cube **3** is removed. Cube **1** is moved to the left and then projected towards the stationary cube **2** with speed u. Suppose that after collision cube **1** moves with speed v_1 and cube **2** moves with speed v_2. See Fig. 1.2.

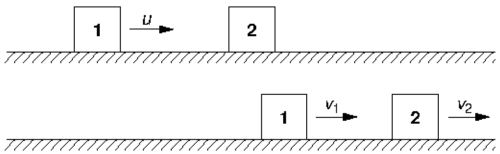


Fig. 1.2

(i) Write down equations for the conservation of momentum and of energy in the elastic collision in terms of m, u, v_1 and v_2.

equation 1 for momentum ....

equation 2 for energy ....[2]

(ii) Put the values $v_1 = 0$ and $v_2 = u$ into your equations in **(i)** to show that they are solutions.

[1]

(c) Cube **3** is replaced in its original position close to cube **2**. Cube **1** is moved to the left and projected towards cube **2** with speed u. See Fig. 1.3.

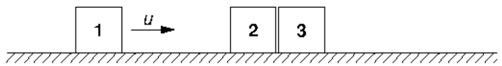


Fig. 1.3

(i) After the collision, cubes **1** and **2** remain at rest and cube **3** moves with speed u. Explain these observations.

....

....

....

....[2]

(ii) Cubes **2** and **3** are now glued together. Describe without calculation or explanation what happens in this situation when cube **1** is projected towards cube **2** as in Fig. 1.3.

....

....

....

....[2]

[Total: 13]

Experimental Physics

3: In an optics experiment, the distance between a mirror and an image is measured eight times, and the following values are obtained.

x/mm:	165	174	170	165	175	167	163	171

(a) Calculate the mean distance $\bar{x}$ from these measurements.

(b) Calculate σ for these measurements explicitly (i.e. calculate the sum of the square deviations of the measurements from the mean, divide by $n-1$, and take the square root).

(c) Calculate σ with your calculator, and check that about two-thirds of the values do lie within the range $\bar{x} \pm \sigma$.

(d) Give the best estimate of the value for the distance, and the appropriate error.

(e) Two further measurements are made of the distance, of 161 mm and 169 mm, giving 10 measurements in total. Calculate the new best estimate for the distance and its error, and comment on this result compared with that in (d).

4: This is a question about combining errors.

(a) Two lengths a and b have been measured and found to be 61.9 ± 0.2 cm and 1.6 ± 0.2 cm.

(i) What are the percentage errors in a and b?

(ii) If $X = a + b$, find the value of X and its error.

(iii) A quantity $Y = a^2 + b^2$. Find the value of Y. Calculate the errors in a^2 and b^2 and hence deduce the error in Y. Which measurement dominates the error here?

(iv) A quantity $Z = a \times b$. Find the value of Z and its error. Which measurement dominates the error here?

(b) In an experiment with a rigid pendulum, the aim is to achieve a value for the acceleration due to gravity, g, with an accuracy of one part in a thousand. The acceleration due to gravity, g, is given by

$$g = k\frac{l}{T^2}$$

where l is the effective length of the pendulum, T its period and k is a numerical constant. You may assume k is known perfectly. Write down an expression relating the error in g to the errors in l and T.

(i) If l can be measured with perfect precision, with what accuracy must T be measured to achieve the required accuracy in g? If T can be measured with perfect precision, with what accuracy must l be measured?

(ii) If $l = 650.50 \pm 0.25$ mm, with what accuracy must T be measured to achieve the required accuracy? If the error in l is doubled, i.e. ± 0.5 mm, what accuracy is then required for T?

5: The current through a diode, I, is measured as a function of the voltage, V, applied across it, and the following values obtained.

voltage / V	current / A
0.30	1.4×10^{-7}
0.35	9.3×10^{-7}
0.40	5.8×10^{-6}
0.45	5.1×10^{-5}
0.50	2.2×10^{-4}
0.55	1.7×10^{-3}
0.60	1.5×10^{-2}
0.65	7.6×10^{-2}
0.70	0.54
0.75	3.6
0.80	12
0.85	25

Plot a straight line graph in Excel to test the theory that

$$I = I_0 \exp\left(\frac{V}{V_0}\right)$$

where I_0 and V_0 are constants, and identify any points which do not fit the theory.
Use least squares regression using Excel to determine the gradient of the graph (excluding any points which you consider are not consistent with the theory), and its error, and hence estimate a value for V_0, and its uncertainty.
[Instructions on how to use Excel to plot a graph and carry out least squares regression can be found in a document entitled *Excel for graphs and calculations* in the Part IA Labs handouts section of the Physics teaching pages: `http://www.phy.cam.ac.uk/students/teaching`.]

6: Observations of the first discovered pulsar, called 'CP1919', were made on 1967 December 24 and 25, by Jocelyn Bell-Burnell. The design of the telescope was such that it could only observe the pulsing signal from the pulsar for a few minutes each day. The data were recorded as pen traces on a paper chart recorder. Regular pulses were usually identifiable on the chart, but at times they were not well-defined or were very weak, and occasionally they were not discernible at all. The times at which individual pulses arrived could be determined from the chart.

An initial estimate of the period of this pulsar obtained from a previous set of observations was (1.337301 ± 0.000004) s. To obtain a more accurate estimate of the period, two well-defined pulses were identified on the chart records, one from each of the two days, 1967 December 24 and 1967 December 25; their arrival times were measured to be as follows:

1967 December 24	$13^{\mathrm{h}}\ 10^{\mathrm{m}}\ 51.8^{\mathrm{s}}$
1967 December 25	$13^{\mathrm{h}}\ 04^{\mathrm{m}}\ 51.0^{\mathrm{s}}$

The accuracy with which each time was determined is about 0.1 s.

(a) Explain why the method of exact fractions is required in this case if we are to obtain a more accurate estimate of the period from these data.

(b) Find the time interval between the two signals; using the method of exact fractions, determine the number of pulses in this interval and hence obtain a revised estimate for the period of the pulsar. From the accuracy with each time was determined, estimate the error in the time interval between the two signals and hence find the error in the new estimate of the period.

[If you are interested, a copy of the original chart records from which these measurements were made can be found in a document entitled *Pulsar chart record* in the Part IA Dynamics handouts section of the Physics teaching pages: `http://www.phy.cam.ac.uk/students/teaching`.]

Forces

7: This question is about the basic concept of a force.

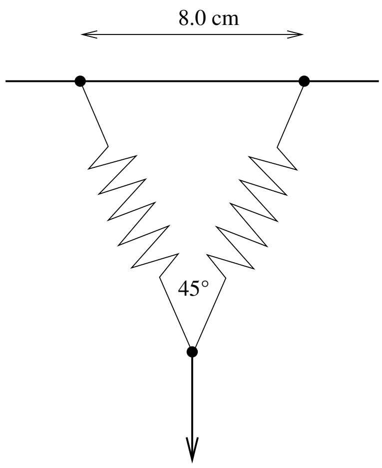
(a) A spring with a natural length $l = 10$ cm is found to extend to a length 11 cm when a mass $m = 0.10$ kg is hung from it in the Earth's gravitational field. Find k, the spring constant (the force per unit extension) of the spring.

(b) Treating the spring as ideal, what would be its extension when a mass $m = 100$ kg is hung from it? In reality, what do you suppose would happen if you tried this?

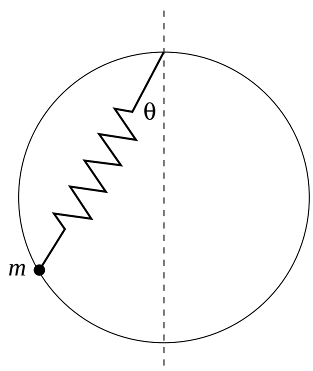
(c) You tie a rope around the trunk of a tree and pull on the rope with a horizontal force of 200 N. What is the tension in the rope? Draw separate 'free-body' diagrams showing the forces on your hands, your feet and on the rope.

(d) You and a friend pull on opposite ends of a rope, each with a force of 200 N. What is the tension in the rope?

(e) Two springs, each identical to that in part (a) are attached to a wall at points 8.0 cm apart. Their other ends are connected together and to a string as shown in the diagram below. The string is pulled perpendicular to the wall, in such a way that the springs are horizontal and make an angle of 45° to each other. What is the tension in the string?



8: A small ring of mass m slides without friction round a circular hoop of radius r in the vertical plane. As shown in the diagram, the mass is connected to the top of the hoop by a spring which has a natural length r, and spring constant k.



(a) Draw a diagram showing the directions of the forces on the ring due to its weight and the tension in the spring.

(b) Show that when the spring makes an angle θ to the vertical, the length of the spring is $2r\cos\theta$.

(c) In addition to the weight of the ring and the tension in the spring, what other force must be acting on the ring for it to be in equilibrium? Show this force on your diagram. (Remember that there is no frictional force between the ring and the hoop.)

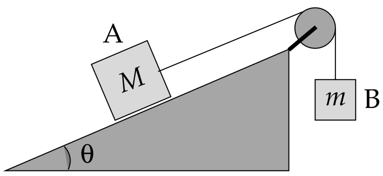
(d) By resolving the components of the weight of the ring and the tension in the spring *in one appropriate direction only*, show that in static equilibrium the angle the spring makes to the vertical is θ_{eq}, where

$$\cos\theta_{\text{eq}} = \frac{1}{2}\frac{1}{(1-(mg/(kr)))}.$$

[You may need the trigonometric identity that $\sin 2\theta = 2\sin\theta\cos\theta$.]

(e) What happens if $mg/(kr) > 0.5$?

9: A block A of mass M on an inclined plane is attached to another block B of mass m via a string passing over a frictionless pulley as shown below. The mass m of block B can be varied. The coefficients of static and dynamic friction between block A and the plane are μ_s and μ_d (where $\mu_d < \mu_s$) and the plane is inclined at an angle θ to the horizontal.



(a) The blocks are stationary. Draw diagrams showing the forces on each block if the mass m of block B is such that block A is on the point of sliding *down* the plane. Hence determine, in terms of M, μ_s and θ, the minimum value of m, $m_{\min}$, for which the system will remain stationary unless disturbed. Comment on the situation when $\tan\theta \leq \mu_s$.

(b) The blocks are still stationary, but now the mass m is such that block A is on the point of sliding *up* the plane. Draw diagrams showing the forces on each block in this case, and hence determine the maximum value of m, $m_{\max}$, for which the system will remain stationary unless disturbed.

(c) Describe in words what happens to the magnitude and direction of the frictional force between block A and the plane as m is increased from $m_{\min}$ to $m_{\max}$. What is the frictional force when $m = M\sin\theta$?

(d) Assume $m = m_{\max}$. As shown above, if the system is stationary then it will remain so unless disturbed. However if block A is briefly disturbed and starts to slide up the plane, it will subsequently *accelerate* up the incline and block B will *accelerate* vertically downwards. Explain, without detailed calculation, why the blocks accelerate under these circumstances.

10: This is a question about the work done by a force in various situations.

(a) A particle moves along the x-axis under the influence of a force F_x in the x-direction, where $F_x = A/x^2$. The particle moves from $x = a$ to $x = b$; find the work done by the force.

(b) A particle moves from a point $\underline{\boldsymbol{r}}_1 = (1.0\hat{\underline{\boldsymbol{i}}} + 2.0\hat{\underline{\boldsymbol{j}}} + 1.0\hat{\underline{\boldsymbol{k}}})$ m to $\underline{\boldsymbol{r}}_2 = (3.0\hat{\underline{\boldsymbol{i}}} - 7.0\hat{\underline{\boldsymbol{j}}} - 1.0\hat{\underline{\boldsymbol{k}}})$ m. Whilst undergoing this displacement it is acted on by a constant force $\underline{\boldsymbol{F}} = (3.0\hat{\underline{\boldsymbol{i}}} + 4.0\hat{\underline{\boldsymbol{j}}})$ N. Show that the work done by the force is -30 J; what does the negative sign indicate here?

(This problem continues on the next page.)

(c) A block of mass m is dragged at a constant speed v along a horizontal floor by a force of magnitude F acting at an angle θ to the horizontal. Draw a diagram to show the forces acting on the block.

(i) Find an expression for the coefficient of dynamic friction.

(ii) Find the work done by the applied force F when the block is displaced through a horizontal distance s. During this process, how much work is done by the vertical forces which act on the block? What happens to the work done by the applied force F?

11: This is a question about using potential energy functions to find equilibrium positions.

(a) The potential energy of an object which can only move along the x-axis is given by

$$U(x) = 2x^2 - x^4.$$

Find all the equilibrium positions and determine whether they are stable or unstable. Sketch the function $U(x)$.

(b) The interatomic interaction between two atoms a distance r apart in a simple molecule is described by the following potential energy function:

$$U(r) = U_0\left(\left(\frac{a}{r}\right)^{12} - 2\left(\frac{a}{r}\right)^{6}\right),$$

where a and U_0 are constants.

(i) Find expressions for the force, $F(r)$, between the atoms, their equilibrium separation and their potential energy at this separation.

(ii) To what values does $U(r)$ tend when $r \to \infty$ and $r \to 0$? What can you deduce from these about the stability of the equilibrium?

(iii) Draw clearly labelled sketches of $U(r)$ and $F(r)$ as functions of r.

Kinematics

12: A gun fires a shell with a fixed speed at the muzzle of 300 $\mathrm{m\,s^{-1}}$, at an angle of elevation which can be varied. Find the angle of elevation that is required to obtain maximum range for a target at the same level as the gun, ignoring the effects of air resistance. Find the corresponding range.

13: A particle falls from rest through a viscous medium. Its speed v at a later time t is given by

$$v = v_\mathrm{t}(1 - \mathrm{e}^{-\alpha t})$$

where v_t is a constant.

(a) Find an equation for the displacement, x, assuming the particle starts from $x = 0$.

(b) Show that the acceleration, a, decays exponentially with time and is also given by

$$a = \alpha(v_\mathrm{t} - v).$$

(c) What is the limiting value of the speed when $t \to \infty$? What is the acceleration when the speed has this limiting value? Is this what you would expect?

(d) Draw clearly labelled sketches of the variations of a, v and x with time t.

Dynamics

14: A diver of mass m begins a descent from a 10-m diving board with zero initial speed.

(a) Calculate the speed v_0 on impact with the water and the approximate elapsed time from dive until impact.

(b) Assuming that under the water the downwards gravity force is exactly balanced by the upwards buoyancy force and $-cv^2$ is the drag force due to the water, the equation of motion for the vertical descent of the diver through the water is given by:

$$ma = -cv^2,$$

where a is the downwards acceleration of the diver. Hence show that the speed v varies with depth x as

$$m\frac{\mathrm{d}v}{\mathrm{d}x} = -cv.$$

(You will need to use the chain rule to find an expression for a in terms of v and x rather than v and t.)

(c) Show by substitution that $v = A\exp(-\alpha x)$ is a solution to this equation and find α; then, by imposing the condition that $v = v_0$ at $x = 0$, find A.

(d) If $c/m = 0.4\ \mathrm{m}^{-1}$, estimate the depth at which $v = 0.1v_0$.

(e) The relationship between time t under water and depth x is given by

$$t = \frac{1}{\alpha A}\left(\mathrm{e}^{\alpha x} - 1\right).$$

By differentiating both sides of the equation with respect to t (implicit differentiation), or otherwise, show that this is consistent with the expression for v in (c). How long does it take for the diver to reach the bottom of a 5-m deep pool?

15: This is a problem about a block sliding up an inclined plane.

(a) A block is projected up a smooth inclined plane. The plane is inclined at an angle θ to the horizontal. The block is given an initial speed v and it slides a distance L up the plane. By considering conservation of energy, find the relationship between v, θ, L and g.

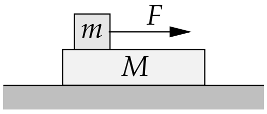
(b) The same block is now projected up a rough inclined plane, also inclined at an angle θ to the horizontal. The block is given the same initial speed v as in (a) and now slides a distance l up the rough plane. By considering the work done against friction, show that the coefficient of dynamic friction, μ_d, is given by

$$\mu_\mathrm{d} = \left(\frac{L}{l} - 1\right)\tan\theta.$$

(c) Determine the condition that, after it reaches its highest point, the block will slide back down the rough plane given that the coefficient of static friction is μ_s.

(d) If the block does slide back down the rough plane, show that the acceleration with which it does so is given by $g\sin\theta(2 - (L/l))$, and that its speed when it gets to the bottom is $\sqrt{2g\sin\theta(2l - L)}$.

16: A slab of mass M rests on a frictionless floor. A block of mass m rests on top of the slab as shown in the diagram. The coefficients of static friction and dynamic friction between the block and the slab are μ_s and μ_d respectively and $\mu_d < \mu_s$. The block is pulled by a horizontal force of magnitude F.



(a) Using Newton's 3rd law, draw free-body diagrams for both the block and the slab to show all the forces acting on them; note in particular that the direction of the frictional force exerted by the slab on the block is determined by considering the direction of the block's impending acceleration.

(b) If the block and slab accelerate together without slipping with a common acceleration, deduce a value for this acceleration. By considering your free-body diagrams, show that the maximum acceleration the slab can have is given by $\mu_s mg/M$. Hence explain why the block and slab will only accelerate at the same rate if F is below a limiting value F_{lim}; show that $F_{\text{lim}} = (M+m)\mu_s mg/M$, where g is the acceleration due to gravity.

(c) For $F > F_{\text{lim}}$, find expressions for the accelerations of the slab and the block in terms of M, m, μ_d, F and g.

(d) If $M = 40$ kg, $m = 10$ kg, $\mu_s = 0.6$, and $\mu_d = 0.4$, what are the accelerations of the slab and the block if (i) $F = 50$ N and (ii) $F = 100$ N?

17: A uniform chain has length l and total mass M and consists of many links. It is held at one end so that it hangs vertically above a table, with the other end just touching the table top. Draw a diagram illustrating the situation.

(a) The chain is released and falls freely. What is now the force between the links? Find an expression for the time T it takes for the topmost link in the chain to reach the table.

(b) Draw a diagram illustrating the situation at time t $(< T)$ after the chain is released. At this time t, what length of chain lies on the table and what is the instantaneous speed of the falling section of the chain? Explain in words why the force on the table at time t is greater than the weight of the part of the chain lying on the table.

(c) Work out the mass Δm which hits the table in the small time interval from t to $t+\Delta t$. Find the change in momentum of Δm when it hits the table and hence show that the instantaneous force on the table due to the deceleration of the links is Mg^2t^2/l .

(d) What is the total force acting on the table as a function of time? Show that the maximum value of the total force is three times the total weight of the chain.

18: The body of a rocket has a mass m_b and also carries initially a mass m_f of fuel.

(a) Use the method described in the lectures to show that the speed u of the rocket after all the fuel is burnt is given by

$$u = u_0 \ln\left(1 + \frac{m_\mathrm{f}}{m_\mathrm{b}}\right),$$

where u_0 is the speed of the ejected gases (assumed constant) relative to the body of the rocket. Be careful about the signs in your derivation, showing clearly how you arrive at the above result.

(b) A rocket of body mass 90 kg carries an initial fuel mass of 300 kg, and is to be used to lift vertically a payload of mass 10 kg to a height of 100 km above the Earth's surface. On the assumption that all the fuel is burnt in a very short time relative to the time of flight of the rocket, what must be the speed of the ejected gases relative to the rocket?

(c) The rocket described in (a) is now divided into two stages, each with body mass $m_\mathrm{b}/2$ and initial fuel mass $m_\mathrm{f}/2$. The first stage is discarded when all its fuel has been consumed. What is now the final speed of the second stage? Explain why this is larger than your answer to part (a).

19: A tennis ball of mass m is travelling towards a wall in a direction perpendicular to the wall; defining the direction of travel to be the positive x-direction, its velocity is given by $\underline{\boldsymbol{v}} = v\widehat{\underline{\boldsymbol{i}}}$. It hits the wall and then rebounds with a velocity $-\underline{\boldsymbol{v}}$. The ball is in contact with the wall for a time τ.

(a) Find expressions for (i) the impulses exerted by the wall on the ball, and by the ball on the wall and (ii) the average force $\underline{\boldsymbol{F}}_\mathrm{average}$ exerted by the wall on the ball during the period of contact.

(b) Write down the change in the kinetic energy of the ball from the time when it first comes into contact with the wall until the time when it comes instantaneously to rest; hence, assuming that a steady force of magnitude F_average is acting, estimate the amount by which the ball is compressed during the collision.

(c) A slightly more realistic model for the force of the wall on the ball — which assumes that the force is proportional to the compression of the ball — is given by

$$\underline{\boldsymbol{F}}(t) = \begin{cases} -F_\mathrm{max}\sin(\pi t/\tau)\,\widehat{\underline{\boldsymbol{i}}} & \text{if } 0 \le t \le \tau\,, \\ 0 & \text{otherwise.} \end{cases}$$

Find an expression for the relationship between F_average and the maximum force F_max exerted by the wall on the ball. Make a labelled sketch of $F(t)$ as a function of t; indicate on your sketch how F_average relates to F_max.

(d) The tennis ball has a mass of 0.057 kg, hits the wall with a speed of 11 km h^{-1} and rebounds with a speed of 11 km h^{-1}; it is in contact with the wall for 6×10^{-3} s. What are the values of F_average and F_max? Use your result from (b) to estimate the amount by which the tennis ball is compressed during the collision; how does this compare with the size of the tennis ball?

[If you are interested, a paper by Rod Cross entitled *The bounce of a ball* describing measurements of the dynamics of bouncing balls can be found in the Part IA Dynamics handouts section of the Physics teaching pages: `http://www.phy.cam.ac.uk/students/teaching`.]

Frames of reference (and collisions)

20: A cyclist travels along a straight road at speed v on two separate occasions; on Day 1 there is no wind, whereas on Day 2 there is a steady wind blowing at a speed w in a direction perpendicular to the road (i.e. a cross-wind). The force on the cyclist due to air resistance is proportional to the *square* of the speed of the air relative to the cyclist; as a consequence, to maintain the same constant speed v on Day 2 as on Day 1, the cyclist has to produce a greater power output. In what follows, assume that $v = 20$ km h^{-1} and $w = 40$ km h^{-1}.

(a) Draw diagrams showing the velocities of the cyclist and air in both the road frame and the cyclist's frame of reference (i) on Day 1 when there is no wind and (ii) on Day 2 when there is the cross-wind. Hence find the speed of the air and the direction in which it is moving relative to the cyclist when the steady cross-wind is blowing.

(b) Draw additional diagrams showing the velocity of the cyclist and the force due to air resistance on Day 1 and on Day 2. Hence deduce the factor by which the cyclist must increase power to maintain a constant speed in the cross-wind.

21: In a road accident at a cross-roads, a car of mass 1.25×10^3 kg travelling at a speed of 40 km h^{-1} from west to east hits a truck of mass 4.0×10^3 kg travelling at a speed of 30 km h^{-1} travelling from north to south. The vehicles lock together and skid off the road.

(a) Draw diagrams showing the situation just before the collision from the points of view of the pilot of a helicopter (i) hovering above the intersection, and (ii) flying parallel to, and keeping pace with, the car before the collision.

(b) In each case, find the direction and speed of the vehicles after the collision and draw these quantities on the diagrams.

(c) Find the total change in kinetic energy in each case. Comment on your answers.

22: A heavy 'superball' of mass M and a very much lighter 'superball' of mass m are dropped together vertically with the light ball directly above the heavy ball, but not quite in contact. (A superball is a rubber ball which is almost perfectly elastic.) Assuming that $M \gg m$ show that the lighter ball will fly up to almost nine times the height from which it was dropped as follows.

(a) Draw two diagrams showing the two balls, not quite touching each other, in the laboratory frame just before and just after the heavy ball hits the ground. Note that the lighter ball is still moving towards the ground in the second diagram.

(b) Draw two more diagrams, both in the frame of reference moving with the heavy ball (i.e. its 'instantaneous rest frame') after it has hit the ground, just before and just after it collides with the light ball.

(c) Transform back to the laboratory frame and use the law of conservation of energy to find the answer.

23: What is the zero-momentum frame, and why is it useful in the description of collisions? An α-particle moving slowly with velocity $\underline{\boldsymbol{u}}$ collides elastically with a stationary proton. As a result of the collision the α-particle is deflected from its initial line of flight through an angle ϕ in the laboratory frame.

(a) Find the velocity of the zero-momentum frame. Deduce the velocities of the particles before the collision in the zero-momentum frame; draw a diagram to illustrate the situation before the collision in the zero-momentum frame.

(b) Deduce the speeds of the particles after the collision in the zero-momentum frame; draw a diagram showing the velocities of the particles after the collision in the zero-momentum frame, assuming that the trajectory of the α-particle deviates from its initial line of flight through an (arbitrary) angle θ in the zero-momentum frame.

(c) Draw a velocity vector diagram to transform back to the laboratory frame. By considering the full range of θ, deduce that ϕ cannot exceed $\sin^{-1}(1/4) = 14^\circ\!.5$.

24: A nucleus of mass 20 m_u ($1m_\mathrm{u} = 1.66\times10^{-27}$ kg) is moving at a speed of 3.0×10^6 m s^{-1}, measured in the laboratory frame, when it breaks into a pair of fragments with an energy release of 1.0×10^{-12} J. This energy appears as extra kinetic energy of the two fragments. The heavier fragment has a mass of 16 m_u and is emitted at 90° to the original line of flight as measured in the laboratory frame. (In the following calculations you may assume that the change in mass which supplies the extra energy is negligible.)

(a) Draw two diagrams showing the situation before and after fragmentation in the laboratory frame.

(b) Draw two diagrams showing the kinematics in the zero-momentum frame, and find the velocities of the fragments in this frame.

(c) Draw a velocity vector diagram to transform back to the laboratory frame. Hence find the velocity (speed and direction) of the lighter fragment in the laboratory frame.

Numerical and some algebraic answers

1: $\alpha = \beta = 1/2$, $\gamma = -5/2$.

2: (2.2) (b) (ii) $-u/3$, $2u/3$.

3: (a) 168.8 mm; (b) 4.4 mm; (c) 4.4 mm again, 5 of 8; (d) (168.8 ± 1.6) mm; (e) (168.0 ± 1.5) mm.

4: (a) (i) 0.3%, 13%; (ii) 63.5 ± 0.3 cm; (iii) $(3.83\pm0.02)\times10^3$ cm^2 or $(3.834\pm 0.025)\times10^3$ cm^2; (iv) 99 ± 12 cm^2; (b) (i) 1 in 2000, 1 in 1000; (ii) ≈ 1 in 2000, ≈ 1 in 3000.

5: $V_0 = (0.0263\pm0.0002)$ V.

6: (b) (1.337300 ± 0.000002) s.

7: (a) 98 N m^{-1}; (b) 10 m; (c) 200 N; (d) 200 N; (e) 0.82 N.

9: (a) $m_\mathrm{min} = M(\sin\theta - \mu_\mathrm{s}\cos\theta)$; (b) $m_\mathrm{max} = M(\sin\theta + \mu_\mathrm{s}\cos\theta)$.

10: (a) $A\left(\dfrac{1}{a}-\dfrac{1}{b}\right)$; (c) (i) $\dfrac{F\cos\theta}{mg - F\sin\theta}$.

11: (a) $x = 0$ (stable), $x = \pm1$ (unstable); (b) (i) $r = a$, $U = -U_0$; (ii) 0, ∞.

12: Elevation 45°, 9.2 km.

13: (a) $x = v_\mathrm{t}\left(t - \frac{1}{\alpha}\left(1-\mathrm{e}^{-\alpha t}\right)\right)$; (c) $v = v_\mathrm{t}$, $a = 0$.

14: (a) $v_0 = 14$ m s^{-1}, $t_0 = 1.43$ s; (c) $\alpha = c/m$, $A = v_0$; (d) 5.8 m; (e) 1.14 s.

15: (a) $v^2 = 2gL\sin\theta$.

16: (c) $\mu_\mathrm{d}mg/M$, $(F/m) - \mu_\mathrm{d}g$; (d) (i) both 1.0 m s^{-2}, (ii) 0.98 m s^{-2}, 6.1 m s^{-2}.

18: (b) 1010 m s^{-1}; (c) $u = u_0\ln\left(1+\dfrac{m_\mathrm{f}}{m_\mathrm{b}}\right) + u_0\ln\left(\dfrac{m_\mathrm{b}+m_\mathrm{f}}{m_\mathrm{b}+\frac{1}{2}m_\mathrm{f}}\right)$.

19: (a) $-2mv\hat{\underline{i}}$, $2mv\hat{\underline{i}}$, $-(2mv/\tau)\hat{\underline{i}}$; (b) $v\tau/4$; (c) $F_\mathrm{average} = (2/\pi)F_\mathrm{max}$; (d) 60 N, 90 N, 5 mm.

20: (a) 44.7 km h^{-1} at 63° to the direction of travel; (b) $\sqrt{5}$.

21: (b) (i) 24.8 km h^{-1}, 157° from north through east; (ii) 38.1 km h^{-1}, 233°; (c) 9.2×10^4 J.

24: (b) 3.88×10^6 m s^{-1}, 1.55×10^7 m s^{-1}; (c) 1.79×10^7 m s^{-1}, $33\overset{\circ}{.}3$ to the original line of flight.

If you find any errors please contact the lecturer (Jeremy Baumberg, jjb12@cam.ac.uk).